Carnot cycle of an ideal paramagnet

Consider the four steps of a Carnot engine with the operating material in the form of an ideal paramagnet. The equation of state is Curie’s law, \( M = DH/T \), where \( H \) is the magnetic field, \( T \) the absolute temperature, and \( D \) a constant. The internal energy is a monotonically increasing function, \( U(T) \), of temperature.

(a) Determine the heat transfer, \( \Delta Q \), the work performance, \( \Delta W \), and the change in internal energy, \( \Delta U \), for each of the four steps:

1 → 2 isothermal demagnetization: \( T = T_H = \text{const}, M_2 < M_1 \).
2 → 3 adiabatic demagnetization: \( S = \text{const}, M_3 < M_2 \).
3 → 4 isothermal magnetization: \( T = T_L = \text{const}, M_4 > M_3 \).
4 → 1 adiabatic magnetization: \( S = \text{const}, M_1 > M_4 \).

(b) Sketch the Carnot cycle in the \((M,H)\)-plane and in the \((U,S)\)-plane.

(c) Show that the efficiency is \( \eta_C = 1 - T_L/T_H \).

Solution: