Reconstructing the equation of state of a fluid system

A fluid system is found to have a thermal expansivity \( \alpha_p = \left( nR/pV \right) + \left( na/RT^2V \right) \) and an isothermal compressibility \( \kappa_T = \left( n/V \right)\left[ Tf(p) + b/p \right] \), where \( a, b \) are constants and \( f(p) \) is an unknown function.

(a) Find the function \( f(p) \) which makes the two response functions thermodynamically consistent.
(b) Reconstruct the equation of state \( V = V(T, p) \) from the two response functions.

Solution: