The mean-field ferromagnet is specified by the heat capacity $C_M = 0$ and by the equation of state $M = \tanh[(H + \lambda M)/T]$, where $\lambda$ is a constant. In zero magnetic field ($H = 0$), this system undergoes a continuous transition at temperature $T_c = \lambda$ between a paramagnetic phase ($M = 0$) and a ferromagnetic phase ($M \neq 0$).

(a) Calculate an analytic expression for the isothermal susceptibility $\chi_T(T, M)$ from the equation of state. Use the numerically determined $M(T, H = 0)$ from [tex45] to plot $\chi_T(T, H = 0)$ versus $T$ for $0 \leq T \leq 2T_c$.

(b) Determine the heat capacity $C_H(T, M) = T\alpha_H^2/(\chi_T - \chi_S)$ from $\alpha_H = (\partial M/\partial T)_H$, $\chi_T = (\partial M/\partial H)_T$, $\chi_S = (\partial M/\partial H)_S$, and plot $C_H$ versus $T$ for $0 \leq T \leq 2T_c$.

(c) Plot in the same diagram (with different symbols) the function $T(\partial S/\partial T)_{H=0}$ by using the data of $S(T, H = 0)$ from [tex45].

Solution: