[tex47] Statistical concept of uncertainty

An experiment has \( n \) possible outcomes that occur with probabilities \( P_1, \ldots, P_n \). The uncertainty about the outcome of the experiment is defined as

\[
\Sigma(P_1, \ldots, P_n) = -\sum_{i=1}^{n} P_i \ln P_i.
\]

(a) Prove that the maximum uncertainty occurs if all \( P_i \) are equal.

(b) The \( n^2 \) combined outcomes of two independent experiments have probabilities \( P_{ij} = P_{i} P_{II} \).

Show that the uncertainty about the combined outcome of the two independent experiments is equal to the sum of the uncertainties of the outcomes of each experiment: \( \Sigma(\{P_{ij}\}) = \Sigma(\{P_{i}\}) + \Sigma(\{P_{II}\}) \).

Solution: