Maxwell distribution in D–dimensional space

The Maxwell velocity distribution of an ideal gas in $D$-dimensional space is

$$f(v) = \left( \frac{m}{2\pi k_B T} \right)^{D/2} e^{-mv^2/2k_B T},$$

where $v = (v_1, \ldots, v_D)$ and $v^2 = v_1^2 + \cdots + v_D^2$. Determine the associated speed distribution $f_S(v)$, the root-mean-square speed $\sqrt{\langle v^2 \rangle}$, the average speed $\langle v \rangle$, and the most frequent speed $v_0$ from $df_S/dv|_{v_0} = 0$.

Solution: