Doppler broadening of atomic spectral lines

Consider a furnace containing a dilute gas at high temperature. Through a small window of the furnace, we observe a particular spectral line of the gas atoms by means of a spectrometer. The width of the observed spectral line is broadened due to the spread of velocities of the gas atoms. This effect is called Doppler broadening. The relativistic Doppler shift of the wavelength is \( \lambda = \lambda_0 \sqrt{(1 + v/c)/(1 - v/c)} \). For the case under consideration we can assume that \( v/c \ll 1 \).

Show that the intensity profile is given by the expression

\[
I(\lambda) \propto \exp\left(-\frac{mc^2(\lambda - \lambda_0)^2}{2\lambda_0^2k_B T}\right),
\]

where \( T \) is the temperature of the furnace, \( c \) is the speed of light, \( m \) is the mass of the gas atoms, and \( \lambda_0 \) is the wavelength of the radiation emitted by an atom at rest.

Solution: