Adiabates of the classical ideal gas

The classical ideal gas is specified by the thermodynamic equation of state $pV = nRT$ and by the internal energy (caloric equation of state) $U = C_V T$ with $C_V = \alpha nR = \text{const}$ [$\alpha = \frac{3}{2}$ (monatomic), $\alpha = \frac{5}{2}$ (diatomic), $\alpha = 3$ (polyatomic)]. A reversible process with $S = \text{const}$ is called isentropic or adiabatic and is characterized by the curve $pV^\gamma = \text{const}$. No heat is exchanged in an adiabatic process: $dU = \delta W$, $\delta Q = 0$. Find $\gamma$ as a function of $\alpha$.

Solution: