Effect of escaping particles on temperature of 1D ideal gas

A classical ideal gas (particle mass $m$, particle density $n$, temperature $T$, energy density $u = \frac{1}{2}nk_B T$) is contained in the region $0 < x < L$. The particles can only move in $\pm x$-direction. We assume that the velocities satisfy a 1D Maxwell distribution at all times. All particles that hit the wall at $x = 0$ are reflected elastically. The wall at $x = L$ allows any particle that hits it to pass through with a probability $\epsilon_0 \ll 1$, independent of the particle’s energy. Otherwise the particle is reflected elastically.

(a) Calculate the rate at which the system loses particles and energy. Express the rates in the form $\frac{dn}{dt} = f_n(n, T)$ and $\frac{du}{dt} = f_u(n, T)$.

(b) The slowly varying particle density $n(t)$ and energy density $u(t)$ cause a slowly varying temperature $T(t)$ of the remaining gas. Derive from the results of (a) a differential equation for the function $T(t)$ and solve it.

Solution: