Classical ideal gas in uniform gravitational field

Consider a column with cross-sectional area $A$ of a classical ideal gas ($N$ atoms of mass $m$) in a uniform gravitational field of magnitude $g$. The gas is in thermal equilibrium at temperature $T$. The Hamiltonian reads:

$$H = \sum_{l=1}^{N} \left( \frac{p_{l}^{2}}{2m} + mgz_{l} \right),$$

where $z_{l}$ is the height of particle $l$ above sea level.

(a) Find the probability density $\rho_{1}(z)$ for the vertical positions of individual gas atoms.

(b) Find the pressure distribution $p(z)$.

Solution: