[tex9] Work extracted from finite heat reservoir in infinite environment

A (finite) heat reservoir with heat capacity \( C = \text{const} \) is initially at temperature \( T_H \) and the (infinite) environment at the lower temperature \( T_0 \). Now the reservoir is connected to the environment by a heat engine, which absorbs an infinitesimal amount of heat \( \delta Q \) per cycle, converts part of it into work \( \delta W \), and dumps the rest into the environment. During each cycle the temperature of the reservoir decreases infinitesimally: \( \delta Q = -CdT \). Determine the maximum amount of work \( \Delta W \) that can be extracted from the reservoir before its temperature has dropped to that of the environment. The fraction of the excess internal energy \( U_{ex} = C(T_H - T_0) \) that can be converted into work is characterized by the quantity \( \Delta W/U_{ex} \). Plot this quantity versus the reduced temperature \( (T_H - T_0)/T_0 \) for \( T_0 < T_H < 3T_0 \). Set \( T_H/T_0 = 1 + \epsilon \) with \( \epsilon \ll 1 \) and find the dependence of \( \Delta W/U_{ex} \) on \( \epsilon \) to leading order.

Solution: