Consider a classical ideal gas \( H_N = \sum_{l=1}^{N} (\frac{p_l^2}{2m}) \) in a box of volume \( V \) in equilibrium with heat and particle reservoirs at temperature \( T \) and chemical potential \( \mu \), respectively.

(a) Show that the grand partition function is
\[
Z = \exp \left( \frac{zV}{\lambda_T^3} \right),
\]
where \( z = \exp(\mu/k_B T) \) is the fugacity, and \( \lambda_T = \sqrt{\frac{\hbar^2}{2\pi mk_B T}} \) is the thermal wavelength.

(b) Derive from \( Z \) the grand potential \( \Omega(T, V, \mu) \), the entropy \( S(T, V, \mu) \), the pressure \( p(T, V, \mu) \), and the average particle number \( \langle N \rangle = N(T, V, \mu) \).

(c) Derive from these expressions the familiar results for the internal energy \( U = \frac{3}{2} N k_B T \), and the ideal gas equation of state \( pV = N k_B T \).

Solution: