[tex97] BE gas in $\mathcal{D}$ dimensions IV: heat capacity at high temperature

The internal energy of the ideal Bose-Einstein gas in $\mathcal{D}$ dimensions and at $T \geq T_c$ is given by the following expression:

$$U = \mathcal{N} k_B T \, \frac{\mathcal{D}}{2} \, \frac{g_{\mathcal{D}/2+1}(z)}{g_{\mathcal{D}/2}(z)}.$$

Use this result to derive the following expression for the heat capacity $C_V = (\partial U/\partial T)_{V\mathcal{N}}$:

$$\frac{C_V}{\mathcal{N} k_B} = \left(\frac{\mathcal{D}}{2} + \frac{\mathcal{D}^2}{4} \right) \frac{g_{\mathcal{D}/2+1}(z)}{g_{\mathcal{D}/2}(z)} - \frac{\mathcal{D}^2}{4} \, \frac{g'_{\mathcal{D}/2+1}(z)}{g'_{\mathcal{D}/2}(z)}.$$

Use the derivative $\partial/\partial T$ of the result $g_{\mathcal{D}/2}(z) = \mathcal{N} \lambda_T^{\mathcal{D}}/V$ with $V = L^{\mathcal{D}}$ to calculate any occurrence of $(\partial z/\partial T)_{V\mathcal{N}}$ in the derivation. Use the recursion relation $z g'_n(z) = g_{n-1}(z)$ for $n \geq 1$ to further simplify the results pertaining to $\mathcal{D} \geq 2$.

Solution: