Ultrarelativistic Bose–Einstein gas

Consider a Bose-Einstein gas with ultrarelativistic one-particle energy $\epsilon_k = c \hbar k = cp$ in the grandcanonical ensemble at temperature $T$ and chemical potential $\mu = 0$.

(a) Show that the one-particle density of states is $D(\epsilon) = (4\pi V / \hbar^3 c^3) \epsilon^2$.

(b) Calculate the pressure $p(T)$, the internal energy $U(T, V)$, and the average number of particles in excited states $N_\epsilon(T, V)$.

(c) Show that the heat capacity is $C_V / k_B = [16\pi^5 / 15\hbar^3 c^3] V (k_B T)^3$.

Solution: