Sound velocity in the classical ideal gas II

Show that the internal energy per unit mass and the enthalpy per unit mass of a classical ideal gas with heat capacity \( C_V = \text{const} \) can be expressed as follows in terms of the sound velocity \( c = \sqrt{(\partial p/\partial \rho)_S} \) and the ratio of heat capacities \( \gamma = C_p/C_V \):

\[
\bar{U} = \bar{U}_0 + \frac{c^2}{\gamma(\gamma - 1)}, \quad \bar{E} = \bar{U}_0 + \frac{c^2}{\gamma - 1},
\]

where \( \bar{U}_0 \) is a constant.

Solution: