Spectral Densities with Unbounded Support [nln102]

Spectral densities with unbounded support are encoded by Δ_k-sequences that grow to infinity as $k \to \infty$. The growth law of the Δ_k-sequence determines the high-frequency decay law of the spectral density [Magnus 1985]:

$$\Delta_k \sim k^\lambda \quad \Rightarrow \quad \Phi_0(\omega) \sim \exp\left(-\omega^{2/\lambda}\right). \tag{1}$$

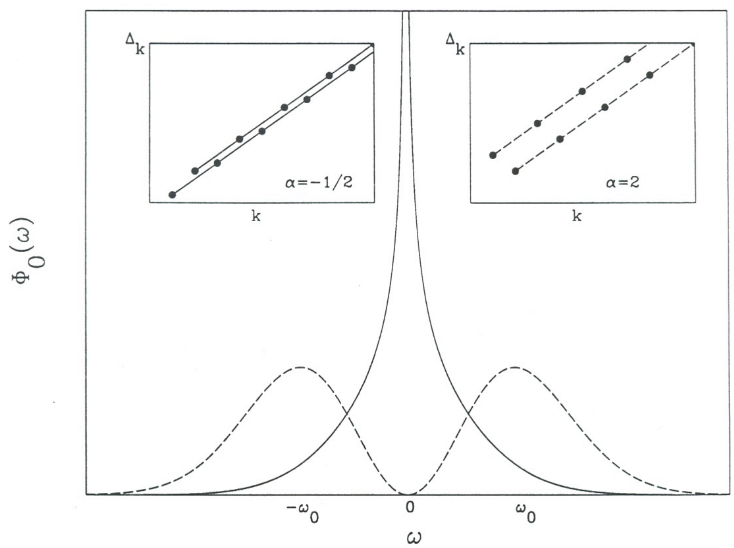
Model spectral density with Gaussian decay and infrared singularity:

$$\Phi_0(\omega) = \frac{2\pi}{\omega_0 \Gamma\big((\alpha+1)/2\big)} \left|\frac{\omega}{\omega_0}\right|^\alpha e^{-(\omega/\omega_0)^2}. \tag{2}$$

Associated Δ_k-sequence has linear growth law. The intercept of the Δ_{2k-1} is governed by the exponent of the infrared singularity.

$$\Delta_{2k-1} = \frac{1}{2}\omega_0^2(2k-1+\alpha), \quad \Delta_{2k} = \frac{1}{2}\omega_0^2(2k). \tag{3}$$

Graphical representations for two cases [Viswanath and Müller 1994]:



Model spectral density with Δ_k-sequence of different growth laws:

$$\Phi_0(\omega) = \frac{2\pi/(\lambda\omega_0)}{\Gamma\big(\lambda(\alpha+1)/2\big)} \left|\frac{\omega}{\omega_0}\right|^\alpha e^{-|\omega/\omega_0|^{2/\lambda}}, \quad M_{2k} = \omega_0^2 \frac{\Gamma\big(\lambda(1+\alpha+2k)/2\big)}{\Gamma\big(\lambda(1+\alpha)/2\big)},$$

where the Δ_k must be determined numerically from the moment M_{2k} as described in [nln85].