Waiting Time Problem [nln11]

Busses arrive more or less randomly at a bus station.
Given is the probability distribution $f(t)$ for intervals between bus arrivals.

Normalization: $\int_0^\infty dt\, f(t) = 1$.

Probability that the interval is larger than t: $P_0(t) = \int_t^\infty dt' f(t')$.

Mean time interval between arrivals: $\tau_B = \int_0^\infty dt\, t f(t) = \int_0^\infty dt P_0(t)$.

Find the probability $Q_0(t)$ that no arrivals occur in a randomly chosen time interval of length t.

First consider the probability $P_0(t'+t)$ for this to be the case if the interval starts at time t' after the last bus arrival. Then average $P_0(t'+t)$ over the range of elapsed time t'.

$$\Rightarrow \; Q_0(t) = c \int_0^\infty dt' P_0(t'+t) \text{ with normalization } Q_0(0) = 1.$$

$$\Rightarrow \; Q_0(t) = \frac{1}{\tau_B} \int_t^\infty dt' P_0(t').$$

Passengers come to the station at random times. The probability that a passenger has to wait at least a time t before the next bus is then $Q_0(t)$:

Probabilty distribution of passenger waiting times:

$$g(t) = -\frac{d}{dt} Q_0(t) = \frac{1}{\tau_B} P_0(t).$$

Mean passenger waiting time: $\tau_P = \int_0^\infty dt\, t g(t) = \int_0^\infty dt Q_0(t)$.

The relationship between τ_B and τ_P depends on the distribution $f(t)$. In general, we have $\tau_P \leq \tau_B$. The equality sign holds for the exponential distribution.