Consider a stationary process $x(t)$. Quantities of interest are expectation values related to $x(t)$.

- Theoretically, we determine ensemble averages: $\langle x(t) \rangle$, $\langle x^2(t) \rangle$, $\langle x(t)x(t+\tau) \rangle$ are independent of $t$.
- Experimentally, we determine time averages: $x(t)$, $x^2(t)$, $x(t)x(t+\tau)$ are independent of $t$.

**Ergodicity**: time averages are equal to ensemble averages.

Implication: the ensemble average of a time average has zero variance.

The consequences for the correlation function

$$C(t_1 - t_2) \doteq \langle x(t_1)x(t_2) \rangle - \langle x(t_1) \rangle \langle x(t_2) \rangle$$

are as follows (set $\tau = t_2 - t_1$ and $t = t_1$):

$$\langle x^2 \rangle - \langle x \rangle^2 = \lim_{T \to \infty} \frac{1}{4T^2} \int_{-T}^{+T} dt_1 \int_{-T}^{+T} dt_2 \left[ \langle x(t_1)x(t_2) \rangle - \langle x(t_1) \rangle \langle x(t_2) \rangle \right]$$

$$= \lim_{T \to \infty} \frac{1}{4T^2} \int_{-2T}^{+2T} d\tau C(\tau)(2T - |\tau|)$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-2T}^{+2T} d\tau C(\tau) \left( 1 - \frac{|\tau|}{2T} \right) = 0.$$ 

**Necessary condition**: $\lim_{\tau \to \infty} C(\tau) = 0$.

**Sufficient condition**: $\int_0^\infty d\tau C(\tau) < \infty$. 

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**Diagram**

- $t_2$
- $t_1$
- $\tau = 2T$
- $\tau = T$
- $\tau = 0$
- $\tau = -T$
- $\tau = -2T$
"T"