Projection operators [nln33]

The relaxation function $c_0(z)$ is determined recursively by a succession of subdivisions of the many-body dynamics into components that are treated rigorously and a remainder that is treated phenomenologically. It is expected that the remainder diminishes in importance as the number of rigorous components is increased systematically.

The time evolution of the dynamical variable $A(t)$ can be conceived as a "pirouette" performed by the vector $|A(t)\rangle$ through the Hilbert space.

The subdivisions are implemented by a sequence of projections onto one-dimensional Hilbert subspaces traversed by $|A(t)\rangle$.

Initial condition: $|f_0\rangle \doteq |A(0)\rangle = |A\rangle$.

Projection operators P_n and $Q_n = 1 - P_n$, $n = 0, 1, \ldots,$.

$$P_0 \doteq |f_0\rangle \frac{1}{\langle f_0|f_0\rangle} \langle f_0|, \qquad P_0^2 = P_0, \qquad P_0^\dagger = P_0, \qquad P_0 Q_0 = Q_0 P_0 = 0.$$

Orthogonal direction:[1]

$$|f_1\rangle = \imath L|f_0\rangle, \quad \langle f_0|f_1\rangle = 0, \quad P_0|f_1\rangle = 0, \quad Q_0|f_1\rangle = |f_1\rangle - P_0|f_1\rangle = |f_1\rangle,$$

$$P_1 = |f_1\rangle \frac{1}{\langle f_1|f_1\rangle} \langle f_1|, \qquad Q_1 = 1 - P_1.$$

The systematic generation of further orthogonal direction will be discussed in the context of the recursion method.

Successive projections filter out particular aspects of the many-body dynamics to be taken into account rigorously. The filters are applied in series. What passes through n filters is the remainder to be treated phenomenologically.

The physical content of this process can be gleaned from the first two projections carried out in detail:

- First projection [nln33],
- Second projection [nln34].

[1] Unitary transformation $e^{\imath Lt}$ makes $\imath LA$ orthogonal to A, implying $\langle A|\imath LA\rangle = 0$.