

Second projection [nl35]

Rewrite memory function from [nl34] with projection operators from [nl33] and apply Dyson identity:

$$\begin{aligned}
 \langle f_0|f_0\rangle\Sigma_1(z) &= \left\langle f_1 \left| \frac{1}{z + \imath L_1} \right| f_1 \right\rangle = \left\langle f_1 \left| \frac{1}{z + \imath L_1 P_1 + \imath L_1 Q_1} \right| f_1 \right\rangle \\
 &= \left\langle f_1 \left| \frac{1}{z + \imath L_1 Q_1} \right| f_1 \right\rangle - \left\langle f_1 \left| \frac{1}{z + \imath L_1 Q_1} \imath L_1 P_1 \frac{1}{z + \imath L_1} \right| f_1 \right\rangle \\
 &= \frac{1}{z} \langle f_1|f_1\rangle - \left\langle f_1 \left| \frac{1}{z + \imath L_1 Q_1} \imath L_1 \right| f_1 \right\rangle \frac{\langle f_0|f_0\rangle}{\langle f_1|f_1\rangle} \Sigma_1(z),
 \end{aligned}$$

where simplifications analogous to [nl34] are carried out.

$$\Rightarrow \Sigma_1(z) = \frac{\langle f_1|f_1\rangle/\langle f_0|f_0\rangle}{z + \frac{1}{\langle f_1|f_1\rangle} \left\langle f_1 \left| \frac{z}{z + \imath L_1 Q_1} \imath L_1 \right| f_1 \right\rangle},$$

$$\begin{aligned}
 \left\langle f_1 \left| \frac{z}{z + \imath L_1 Q_1} \imath L_1 \right| f_1 \right\rangle &= \dots = \left\langle f_1 \left| (-\imath) L_1 Q_1 \frac{1}{z + \imath Q_1 L_1 Q_1} Q_1 \imath L_1 \right| f_1 \right\rangle \\
 &= \left\langle f_2 \left| \frac{1}{z + \imath L_2} \right| f_2 \right\rangle,
 \end{aligned}$$

with $|f_2\rangle = Q_1 \imath L_1 |f_1\rangle$, $L_2 = Q_1 L_1 Q_1$.

Memory function (first termination function) after second projection expressed via second termination function:

$$\Sigma_1(z) = \frac{\Delta_1}{z + \Sigma_2(z)}, \quad \Sigma_2(z) = \frac{1}{\langle f_1|f_1\rangle} \left\langle f_2 \left| \frac{1}{z + \imath L_2} \right| f_2 \right\rangle$$

with continued-fraction coefficients $\Delta_1 = \langle f_1|f_1\rangle/\langle f_0|f_0\rangle$.

The n^{th} projection yields

$$\Sigma_{n-1}(z) = \frac{\Delta_{n-1}}{z + \Sigma_n(z)}, \quad \Sigma_n(z) = \frac{1}{\langle f_{n-1}|f_{n-1}\rangle} \left\langle f_n \left| \frac{1}{z + \imath L_n} \right| f_n \right\rangle$$

with $\Delta_{n-1} = \langle f_{n-1}|f_{n-1}\rangle/\langle f_{n-2}|f_{n-2}\rangle$
and $|f_n\rangle = Q_{n-1} \imath L_{n-1} |f_{n-1}\rangle$, $L_n = Q_{n-1} L_{n-1} Q_{n-1}$.