Energy transfer [nln38]

Hamiltonian of system and interaction with radiation field:

$$\mathcal{H}(t) = \mathcal{H}_0 + \mathcal{H}_1(t) = \mathcal{H}_0 - a(t)A.$$

Interaction between system and radiation field involves energy transfer.

Rate at which average energy of system changes:

$$\frac{d}{dt}\langle\mathcal{H}_0\rangle = \frac{1}{\imath\hbar}\langle[\mathcal{H}_0, \mathcal{H}(t)]\rangle = -\frac{1}{\imath\hbar}a(t)\langle[\mathcal{H}_0, A(t)]\rangle.$$

Calculate linear response $\langle[\mathcal{H}_0, A(t)]\rangle - \underbrace{\langle[\mathcal{H}_0, A]\rangle_0}_{0}$.[1]

Application of Kubo formula [nln27]:

$$\langle[\mathcal{H}_0, A(t)]\rangle = \frac{\imath}{\hbar}\int_{-\infty}^{t} dt' a(t')\langle[[\mathcal{H}_0, A(t)], A(t')]\rangle_0.$$

$$\begin{aligned}
\Rightarrow\ \frac{d}{dt}\langle\mathcal{H}_0\rangle &= -\frac{1}{\hbar^2}a(t)\int_{-\infty}^{t} dt' a(t')\langle[\overbrace{[\mathcal{H}_0, A(t)]}^{-\imath\hbar dA/dt}, A(t')]\rangle_0 \\
&= \frac{\imath}{\hbar}a(t)\int_{-\infty}^{t} dt' a(t')\frac{\partial}{\partial t}\langle[A(t), A(t')]\rangle_0 \\
&= \int_{-\infty}^{+\infty} dt' a(t)a(t')\frac{\partial}{\partial t}\tilde{\chi}_{AA}(t-t')
\end{aligned}$$

with response function

$$\tilde{\chi}_{AA}(t-t') = \frac{\imath}{\hbar}\theta(t-t')\langle[A(t), A(t')]\rangle_0.$$

The time-averaged energy transfer depends only on the absorptive part, $\chi''_{AA}(\omega)$, of the generalized susceptibility as demonstrated in [nex64] for a monochromatic perturbation.

[1] We have $\langle[\mathcal{H}_0, A]\rangle_0 = \mathrm{Tr}\{e^{-\beta\mathcal{H}_0}\mathcal{H}_0 A - e^{-\beta\mathcal{H}_0}A\mathcal{H}_0\}/Z_0 = 0$ in thermal equilibrium.