Elements of Set Theory [nln4]

A **set** S is a collection of **elements** ϵ_i: $S = \{\epsilon_1, \epsilon_2, \ldots\}$.

All elements of **subset** A are also elements of S: $A \subset S$.

The **empty set** $\emptyset$ contains no elements.

If S contains n elements then the number of subsets is 2^n.

Hierarchy of subsets: $\emptyset \subset A \subset A \subset S$.
Transitivity: If $C \subset B$ and $B \subset A$ then $C \subset A$.
Equality: $A = B$ iff $A \subset B$ and $B \subset A$.

Union: $C = A + B$ or $C = A \cup B$.
All elements of S that are contained in A or in B or in both.

Intersection: $C = AB$ or $C = A \cap B$.
All elements of S that are contained in A and in B.

Union and intersection are **commutative**, **associative**, and **distributive**:
$A + B = B + A, \quad AB = BA;$
$(A + B) + C = A + (B + C), \quad (AB)C = A(BC);$
$A(B + C) = AB + AC.$

Some consequences:
$A + A = A, \quad A + \emptyset = A, \quad A + S = S; \quad AA = A, \quad A\emptyset = \emptyset, \quad AS = A.$

Mutually exclusive subsets have no common elements: $AB = \emptyset$.

Partition $\mathcal{P} = [A_1, A_2, \ldots]$ of S into mutually exclusive subsets:
$S = A_1 + A_2 + \ldots$ with $A_iA_j = \emptyset$ for $i \neq j$.

The **complement** $\bar{A}$ of subset A has all elements of S that are not in A.

Some consequences:
$A + \bar{A} = S, \quad A\bar{A} = \emptyset, \quad \bar{\bar{A}} = A, \quad \bar{S} = \emptyset, \quad \bar{\emptyset} = S.$

DeMorgan's law: $\overline{A + B} = \bar{A}\bar{B}, \quad \overline{AB} = \bar{A} + \bar{B}.$

Duality principle:
A set identity is preserved if all sets are replaced by their complements.