

Elementary Probabilities [nl42]

Probabilities can be defined (i) axiomatically, (ii) from relative frequencies of events, or (iii) by assignment to elementary events based on one of two criteria [e.g. Papoulis 1991, Sec. 1-2]:

- symmetry (somewhat circularly),
- principle of insufficient reason (somewhat dubiously).

Ambiguities may arise if the range of events is

- discrete and infinite,
- continuous (such as in Bertrand's paradox [nl41]).

In general, probability densities change under a transformation of variables. Particular assignments are often hard to justify.

In classical equilibrium statistical mechanics, uniform probability densities are assigned to canonical coordinates. This choice is most readily justifiable in the case of action-angle coordinates. Canonical transformations leave probability densities invariant. However, the existence of action-angle coordinates is limited to integrable systems. Only a tiny minority of many-body systems are integrable.