Elements of Probability Theory

Transcription from set theory:

- set → sample space
- subset → event
- element → elementary event

Sample space $S$: set of all possible outcomes in an experiment.
Event $A \subset S$: possible outcome of experiment.

Probability axioms [Kolmogorov 1933]:

- $P(A) \geq 0$ for all $A \subset S$,
- $P(S) = 1$,
- $P(A + B) = P(A) + P(B)$ if $AB = \emptyset$.

Some immediate consequences [nex94]

- $P(\emptyset) = 0$,
- $P(\overline{A}) = 1 - P(A)$,
- $P(A + B) = P(A) + P(B) - P(AB)$.

Under idealized circumstances the sample space is divisible into elementary, mutually exclusive, events to which equal probabilities can be assigned for reasons of symmetry:

$$S = A_1 + \cdots + A_N \text{ with } A_i A_j = \emptyset \text{ for } i \neq j \text{ and } P(A_i) = \frac{1}{N}.$$