Probability Distribution [nln46]

Experiment represented by events in a sample space: $S = \{A_1, A_2, \ldots\}$.

Measurements represented by stochastic variable: $X = \{x_1, x_2, \ldots\}$.

Maximum amount of information experimentally obtainable is contained in the probability distribution:

$$P_X(x_i) \geq 0, \quad \sum_i P_X(x_i) = 1.$$

Partial information is contained in moments,

$$\langle X^n \rangle = \sum_i x_i^n P_X(x_i), \quad n = 1, 2, \ldots,$$

or cumulants (as defined in [nln47]),

- $\langle\langle X \rangle\rangle = \langle X \rangle$ (mean value)
- $\langle\langle X^2 \rangle\rangle = \langle X^2 \rangle - \langle X \rangle^2$ (variance)
- $\langle\langle X^3 \rangle\rangle = \langle X^3 \rangle - 3\langle X \rangle \langle X^2 \rangle + 2\langle X \rangle^3$

The variance is the square of the standard deviation: $\langle\langle X^2 \rangle\rangle = \sigma_X^2$.

For continuous stochastic variables we have

$$P_X(x) \geq 0, \quad \int dx P_X(x) = 1, \quad \langle X^n \rangle = \int dx\, x^n P(x).$$

In the literature $P_X(x)$ is often named 'probability density' and the term 'distribution' is used for

$$F_X(x) = \sum_{x_i < x} P_X(x_i) \quad \text{or} \quad F_X(x) = \int_{-\infty}^{x} dx' P_X(x')$$

in a cumulative sense.