Transformation of Random Variables [nln49]

Consider two random variables X and Y that are functionally related:

$$Y = F(X) \quad \text{or} \quad X = G(Y).$$

If the probability distribution for X is known then the probability distribution for Y is determined as follows:

$$P_Y(y)\Delta y = \int_{y<f(x)<y+\Delta y} dx P_X(x)$$

$$\Rightarrow \;\; P_Y(y) = \int dx P_X(x)\delta\big(y - f(x)\big) = P_X\big(g(y)\big)\,|g'(y)|\,.$$

Consider two random variables X_1, X_2 with a joint probability distribution

$$P_{12}(x_1, x_2).$$

The probability distribution of the random variable $Y = X_1 + X_2$ is then determined as

$$P_Y(y) = \int dx_1 \int dx_2\, P_{12}(x_1, x_2)\delta(y - x_1 - x_2) = \int dx_1 P_{12}(x_1, y - x_1),$$

and the probability distribution of the random variable $Z = X_1X_2$ as

$$P_Z(z) = \int dx_1 \int dx_2\, P_{12}(x_1, x_2)\delta(z - x_1x_2) = \int \frac{dx_1}{|x_1|} P_{12}(x_1, z/x_1).$$

If the two random variables X_1, X_2 are statistically independent we can substitute $P_{12}(x_1, x_2) = P_1(x_1)P_2(x_2)$ in the above integrals.

Applications:

- ▷ Transformation of statistical uncertainty [nex24]
- ▷ Chebyshev inequality [nex6]
- ▷ Robust probability distributions [nex19]
- ▷ Statistically independent or merely uncorrelated? [nex23]
- ▷ Sum and product of uniform distributions [nex96]
- ▷ Exponential integral distribution [nex79]
- ▷ Generating exponential and Lorentzian random numbers [nex80]
- ▷ From Gaussian to exponential distribution [nex8]
- ▷ Transforming a pair of random variables [nex78]