Degrees of Memory [nln51]

Identification of three types of stochastic processes.

The following time ordering is assumed: $t_1 \geq t_2 \geq \cdots \geq \bar{t}_1 \geq \bar{t}_2 \geq \cdots$.

1. **Future independent of present and past.**
   Completely factorizing process.
   $P(x_1, t_1; x_2, t_2; \ldots | \bar{x}_1, \bar{t}_1; \bar{x}_2, \bar{t}_2, \ldots) = P(x_1, t_1) P(x_2, t_2) \cdots$.

   Example: Gaussian white noise: $P(x,t) = (2\pi\sigma^2)^{-1/2} e^{-x^2/2\sigma^2}$,
   $\langle X(t) \rangle = 0, \quad \langle X(t) X(t') \rangle = \sigma^2 \delta(t - t')$,
   $\int d\tau \langle X(t) X(t+\tau) \rangle e^{i\omega\tau} = \sigma^2 = \text{const} \quad$ (spectral density).

2. **Future dependent on present only.**
   Markov process.
   $P(x_1, t_1; x_2, t_2; \ldots | \bar{x}_1, \bar{t}_1; \bar{x}_2, \bar{t}_2, \ldots) = P(x_1, t_1; x_2, t_2; \ldots | \bar{x}_1, \bar{t}_1)$.

3. **Future dependent on present and past.**
   Non-Markovian process.
   $P(x_1, t_1; x_2, t_2; \ldots | \bar{x}_1, \bar{t}_1; \bar{x}_2, \bar{t}_2, \ldots)$.

Comments:

- Type-2 processes are the main focus in parts 6 and 7 of this course.
- Connections discussed in part 8 of this course: (i) type-1 and type-2 processes interlinked in Langevin equation, (ii) type-2 and type-3 processes interlinked in generalized Langevin equation.
- The same physical process may be described as a type-2 process or a type-3 process depending on the level of description and the choice variables.