Markovian or Non-Markovian II [nln53]

Consider a dilute classical gas, i.e. a physical ensemble of free massive particles moving in a box. The particles move with constant velocity between collisions. The average time between collisons is τ_c. Here we focus on the motion of the particles in x-direction.

Short time intervals: $t \ll \tau_c$ (no collisions during any time interval)

(i) Probability distribution of two-component random variable $[x, \dot{x}]$:

$$P([x_1, \dot{x}_1], t_1 | [x_0, \dot{x}_0], t_0) = \delta\big(\dot{x}_1 - \dot{x}_0\big)\delta\big(x_1 - x_0 - \dot{x}_0(t_1 - t_0)\big).$$

The motion is deterministic. The conditional probability distribution is sharp. The process thus described is Markovian. Any additional condition $[x_{-1}, \dot{x}_{-1}]$ associated with a prior time t_{-1} is redundant.

(ii) Probability distribution of one-component random variable x:

(a) If we insist on a Markovian description, by means of the conditional probability distribution,

$$P_1(x_1, t_1 | x_0, t_0),$$

we obtain a broad distribution even though the process is deterministic.

(b) If we insist on a sharp distribution we must choose a non-Markovian description, by means of the probability distribution with two conditions,

$$P_2(x_1, t_1 | x_0, t_0; x_{-1}, t_{-1}).$$

Any further condition x_{-2} associated with a prior time t_{-2} is again redundant.

The contraction of the level of description from $[x, \dot{x}]$ as in (i) to x as in (ii) of one and the same deterministic time evolution shifts information about the process into memory (cf. [nln15]).

Long time intervals: $t \gg \tau_c$ (many collisions during every time interval)

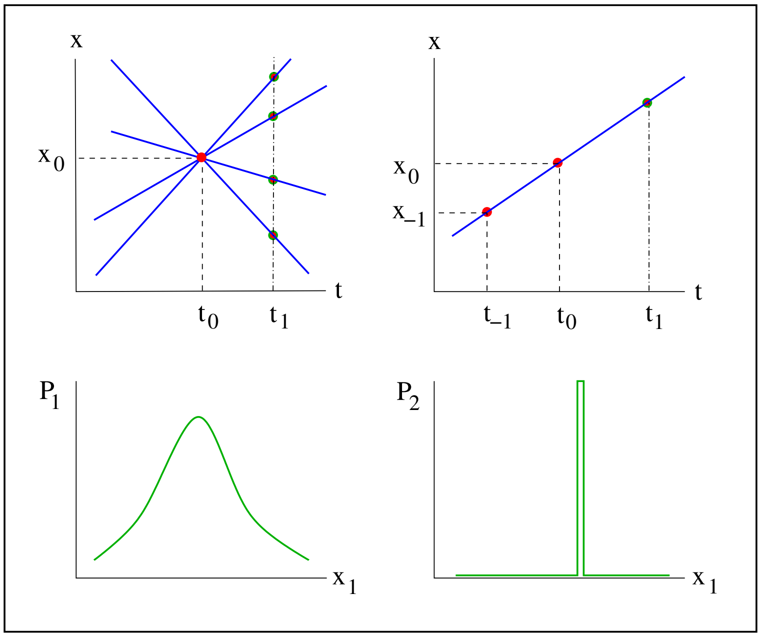
(iii) Markov process $P_1(x_1, t_1 | x_0, t_0)$ is probabilistic (not deterministic).

(iv) Non-Markov process $P_2(x_1, t_1 | x_0, t_0; x_{-1}, t_{-1})$ is also probabilistic.

Both conditional probability distributions are broad. The second condition narrows P_2 down relativ to P_1 if $t_0 - t_{-1}$ is short. With increasing $t_0 - t_{-1}$ the effect of the second condition fades away.

[Illustrations on next page]

Short time intervals: $t \ll \tau_c$ (no collisions during any time interval)



Long time intervals: $t \gg \tau_c$ (many collisions during every time interval)