Random Walk in One Dimension [nln60]

In the following, we analyze this very common process in different ways and also examine variations of it.

- **Walker takes units steps in unit time.** [nex34]
  Position $x_n = n\ell$ of walker at time $t = N\tau$.
  Conditional probability: $P(x_n, t_N|0, 0)$.
  Chapman-Kolmogorov equation (discretized version).
  Binomial distribution.

- **Walker takes smaller steps more frequently.** [nex100]
  Steps left or right with probabilities p and q, respectively.
  Limit: $\ell \to 0, \tau \to 0, p - q \to 0$ with $\ell^2/2\tau = D$ and $(p - q)\ell/\tau = v$.
  Fokker-Planck equation with constant drift and diffusion.

- **Walker's destination drifts and diffuses** [nex101]
  Solution of Fokker-Planck equation via characteristic equation.
  Gaussian with drifting peak and runaway broadening.

- **Walker takes discrete steps randomly in continuous time.** [nex33]
  Master equation for discrete random variable.
  Walker takes step left or right with equal probability.
  Mean time interval between steps: τ.
  Position distributed via modified Bessel function.
  Gaussian function in the limit $\ell \to 0, \tau \to 0$ with $\ell^2/2\tau = D$.

- **Walk that is random only in time.** [nex25]
  Poisson process.
  Walker takes one step in time τ on average (same direction).
  Master equation solved via characteristic equation.
  Poisson distribution rises as a power law and fades out exponentially.

- **Random in Las Vegas.** [nex40]
  Gambling is a biased random walk near a precipice.
  First bias: the odds favor the casino.
  Second bias: the casino has more resources.
  Third bias: the gambler imagines Markovian features.