Fourier's Law for Heat Conduction [nln69]

Heat conduction inside a solid involves three field quantities:

- energy density: $\epsilon(x,t)$,
- heat current: $J(x,t)$,
- local temperature: $T(x,t)$.

Relations between field quantities:

(a) conservation law: $\dfrac{\partial}{\partial t}\,\epsilon(x,t) = -\dfrac{\partial}{\partial x}\,J(x,t)$ (continuity equation),

(b) constitutive law: $J(x,t) = -\lambda\dfrac{\partial}{\partial x}\,T(x,t)$ (Fourier's law),

(c) thermodynamic relation: $\epsilon(x,t) = c_V T(x,t)$.

Material constants:

- specific heat: c_V,
- thermal conductivity: λ,
- thermal diffusivity: $D_T = \lambda/c_V$.

Diffusion equation from (a)-(c):

$$\frac{\partial}{\partial t}\,T(x,t) = D_T\frac{\partial^2}{\partial x^2}\,T(x,t).$$

Applications:

▷ Temperature profile inside wall [nex117]