**Shot Noise**

Electric current in a vacuum tube or solid state device described as a random sequence of discrete events involving microscopic charge transfer:

\[ I(t) = \sum_{k} F(t - t_k). \]

Assumptions:
- uniform event profile \( F(t) \) characteristic of process (e.g. as sketched),
- event times \( t_k \) randomly distributed,
- average number of events per unit time: \( \lambda \).

Attributes characteristic of Poisson process:
- probability distribution: \( P(n, t) = e^{-\lambda t}(\lambda t)^n/n! \),
- mean and variance: \( \langle\langle n\rangle\rangle = \langle\langle n^2\rangle\rangle = \lambda t \).

Probability that \( n \) events have taken place until time \( t \) reinterpreted as probability that stochastic variable \( N(t) \) assumes value \( n \) at time \( t \):

\[ P(n, t) = \text{prob}\{N(t) = n\}. \]

Sample path of \( N(t) \) and its derivative:

\[ N(t) = \sum_{k} \theta(t - t_k), \quad \mu(t) = \frac{dN}{dt} = \sum_{k} \delta(t - t_k). \]

Sample path of electric current:

\[ I(t) = \sum_{k} F(t - t_k) = \int_{-\infty}^{+\infty} dt' F(t - t') \mu(t'). \]
Event profile: $F(t) = q e^{-\alpha t} \theta(t)$ with charge $q_0 = q/\alpha$ per event.

Electric current: $I(t) = \int_{-\infty}^{t} dt' q e^{-\alpha(t-t')} \mu(t')$.

Stochastic differential equation:

$$\frac{dI}{dt} = -\alpha I(t) + q \mu(t). \quad (1)$$

Attributes of Poisson process: $\langle (dN(t)) \rangle = \langle (dN(t))^2 \rangle = \lambda dt$.

Fluctuation variable: $d\eta(t) = dN(t) - \lambda dt \quad \Rightarrow \langle d\eta(t) \rangle = 0$, $\langle [d\eta(t)]^2 \rangle = \lambda dt$.

Average current from (1):\(^1\)

$$dI(t) = [\lambda q - \alpha I(t)] dt + q d\eta(t) \quad \Rightarrow \langle dI(t) \rangle = [\lambda q - \alpha \langle I(t) \rangle] dt$$

$$\Rightarrow \frac{d}{dt} \langle I(t) \rangle = \lambda q - \alpha \langle I(t) \rangle. \quad (2)$$

Current fluctuations from (1) and (2):\(^2\)

$$dI^2 \doteq (I + dI)^2 - I^2 = 2IdI + (dI)^2,$$

$$\langle dI^2 \rangle = 2 \langle I([\lambda q - \alpha I] dt + q d\eta) \rangle + \langle ([\lambda q - \alpha I] dt + q d\eta)^2 \rangle$$

$$= (2\lambda q \langle I \rangle - 2\alpha \langle I^2 \rangle + \lambda q^2) dt + O([dt]^2)$$

$$\Rightarrow \frac{1}{2} \frac{d}{dt} \langle I^2 \rangle = \lambda q \langle I \rangle - \alpha \langle I^2 \rangle + \frac{1}{2} \lambda q^2. \quad (3)$$

Steady state: $\frac{d}{dt} \langle I \rangle_S = 0$, $\frac{d}{dt} \langle I^2 \rangle_S = 0$:

$$\Rightarrow \langle I \rangle_S = \frac{\lambda q}{\alpha}, \quad \langle I^2 \rangle_S \doteq \langle I^2 \rangle_S - \langle I \rangle_S^2 = \frac{q^2 \lambda}{2\alpha}. \quad (4)$$

Applications:

\(\triangleright\) Campbell processes [nex37]

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\(^1\)Use $q \mu(t) dt = q dN(t) = q d\eta(t) + q \lambda dt$.

\(^2\)Use $\langle (I(t) d\eta(t)) \rangle = 0$.