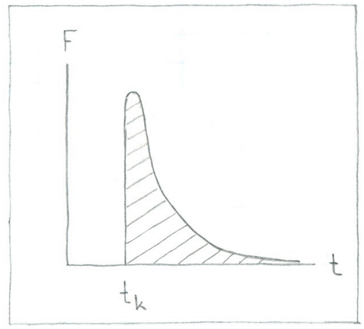
Shot Noise [nln70]

Electric current in a vacuum tube or solid state device described as a random sequence of discrete events involving microscopic charge transfer:

$$I(t) = \sum_k F(t - t_k).$$



Assumptions:

- uniform event profile $F(t)$ characteristic of process (e.g. as sketched),
- event times t_k randomly distributed,
- average number events per unit time: λ.

Attributes characteristic of Poisson process: [nex25] [nex16]

- probability distribution: $P(n,t) = e^{-\lambda t}(\lambda t)^n/n!$,
- mean and variance: $\langle\langle n\rangle\rangle = \langle\langle n^2\rangle\rangle = \lambda t$.

Probability that n events have taken place until time t reinterpreted as probability that stochastic variable $N(t)$ assumes value n at time t:

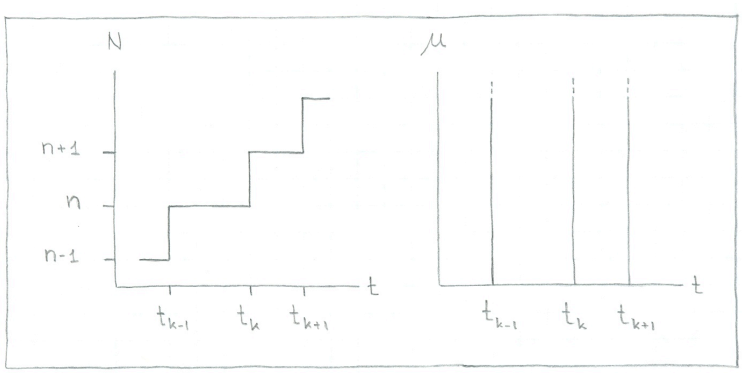
$$P(n,t) = \mathrm{prob}\{N(t) = n\}.$$

Sample path of $N(t)$ and its derivative:

$$N(t) = \sum_k \theta(t - t_k), \quad \mu(t) \doteq \frac{dN}{dt} = \sum_k \delta(t - t_k).$$

Sample path of electric current:

$$I(t) = \sum_k F(t - t_k) = \int_{-\infty}^{+\infty} dt' F(t - t')\mu(t').$$



Event profile: $F(t) = q\, e^{-\alpha t}\theta(t)$ with charge $q_0 = q/\alpha$ per event.

Electric current: $I(t) = \displaystyle\int_{-\infty}^{t} dt' q\, e^{-\alpha(t-t')}\mu(t')$.

Stochastic differential equation:

$$\frac{dI}{dt} = -\alpha I(t) + q\mu(t). \tag{1}$$

Attributes of Poisson process: $\langle\langle dN(t)\rangle\rangle = \langle\langle [dN(t)]^2\rangle\rangle = \lambda dt$.

Fluctuation variable: $d\eta(t) = dN(t) - \lambda dt \quad \Rightarrow \langle d\eta(t)\rangle = 0,\ \langle [d\eta(t)]^2\rangle = \lambda dt$.

Average current from (1):[1]

$$dI(t) = \big[\lambda q - \alpha I(t)\big]dt + q\, d\eta(t) \quad \Rightarrow \langle dI(t)\rangle = \big[\lambda q - \alpha\langle I(t)\rangle\big]dt$$

$$\Rightarrow \frac{d}{dt}\langle I(t)\rangle = \lambda q - \alpha\langle I(t)\rangle. \tag{2}$$

Current fluctuations from (1) and (2):[2]

$$dI^2 \doteq (I + dI)^2 - I^2 = 2IdI + (dI)^2,$$

$$\begin{aligned}\langle dI^2\rangle &= 2\left\langle I\big([\lambda q - \alpha I]dt + qd\eta\big)\right\rangle + \left\langle \big([\lambda q - \alpha I]dt + qd\eta\big)^2\right\rangle \\ &= \big(2\lambda q\langle I\rangle - 2\alpha\langle I^2\rangle + \lambda q^2\big)dt + \mathrm{O}\big([dt]^2\big)\end{aligned}$$

$$\Rightarrow \frac{1}{2}\frac{d}{dt}\langle I^2\rangle = \lambda q\langle I\rangle - \alpha\langle I^2\rangle + \frac{1}{2}\lambda q^2. \tag{3}$$

Steady state: $\dfrac{d}{dt}\langle I\rangle_S = 0, \quad \dfrac{d}{dt}\langle I^2\rangle_S = 0$:

$$\Rightarrow \langle I\rangle_S = \frac{\lambda q}{\alpha}, \quad \langle\langle I^2\rangle\rangle_S \doteq \langle I^2\rangle_S - \langle I\rangle_S^2 = \frac{q^2\lambda}{2\alpha}. \tag{4}$$

Applications:

▷ Campbell processes [nex37]

[1]Use $q\mu(t)dt = qdN(t) = qd\eta(t) + q\lambda dt$.

[2]Use $\langle I(t)d\eta(t)\rangle = 0$.