Consider a set of $N$ independent experiments, each having two possible outcomes occurring with given probabilities.

<table>
<thead>
<tr>
<th>events</th>
<th>$A + B = S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>probabilities</td>
<td>$p + q = 1$</td>
</tr>
<tr>
<td>random variables</td>
<td>$n + m = N$</td>
</tr>
</tbody>
</table>

**Binomial distribution:**

$$P_N(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}.$$  
Mean value: $\langle n \rangle = Np$.  
Variance: $\langle \langle n^2 \rangle \rangle = Npq$.  

In the following we consider two different asymptotic distributions in the limit $N \to \infty$.

**Poisson distribution:**

Limit #1: $N \to \infty$, $p \to 0$ such that $Np = \langle n \rangle = a$ stays finite [nex15].  
$$P(n) = \frac{a^n}{n!} e^{-a}.$$  
Cumulants: $\langle\langle n^m \rangle \rangle = a$.  
Factorial cumulants: $\langle\langle n^m \rangle \rangle_f = a \delta_{m,1}$.  
Single parameter: $\langle n \rangle = \langle\langle n^2 \rangle \rangle = a$.

**Gaussian distribution:**

Limit #2: $N \gg 1$, $p > 0$ with $Np \gg \sqrt{Npq}$.  
$$P_N(n) = \frac{1}{\sqrt{2\pi\langle\langle n^2 \rangle \rangle}} \exp\left(-\frac{(n - \langle n \rangle)^2}{2\langle\langle n^2 \rangle \rangle}\right).$$  
Derivation: DeMoivre-Laplace limit theorem [nex21].  
Two parameters: $\langle n \rangle = Np$, $\langle\langle n^2 \rangle \rangle = Npq$.  
Special case of central limit theorem [nhn9].