Representations of Recursion Method [nln81]

Object of interest: time-dependent correlation function $\langle A(t)A\rangle$.

Recursive part of method: orthogonal expansion of carrier of time evolution.

Heisenberg picture:

- time evolution prescribed by Heisenberg (or Hamilton) equation,
- time evolution carried by dynamical variable,
- Liouvillian operator generates new directions.

Schrödinger picture:

- time evolution prescribed by Schrödinger equation,
- time evolution carried by wave function,
- Hamiltonian operator generates new directions.

Liouvillian representation	**Hamiltonian representation**
$\frac{d}{dt}A(t) = \imath L_{\mathrm{qu}}A(t) = \frac{\imath}{\hbar}[\mathcal{H}, A(t)]$ $\frac{d}{dt}A(t) = \imath L_{\mathrm{cl}}A(t) = -\{\mathcal{H}, A(t)\}$	$\imath\hbar\frac{d}{dt}\|\psi(t)\rangle = \mathcal{H}\|\psi(t)\rangle$
$A(t) = e^{\imath Lt}A(0) = \sum_{k=0}^{\infty} C_k(t)\|f_k\rangle$	$\|\psi(t)\rangle = e^{-\imath\mathcal{H}t/\hbar}\|\psi(0)\rangle = \sum_{k=0}^{\infty} D_k(t)\|f_k\rangle$
$\|f_0\rangle = A(0), \quad \|f_{k+1}\rangle = \imath L\|f_k\rangle - \ldots$	$\|f_0\rangle = \|\psi(0)\rangle = A\|\phi_0\rangle, \quad \|f_{k+1}\rangle = \mathcal{H}\|f_k\rangle - \ldots$
$\tilde{\Phi}(t) = \langle [A(t), A(0)]_+\rangle - \langle A\rangle^2$	$\tilde{S}(t) = \langle A(t)A(0)\rangle - \langle A\rangle^2$
$\mathrm{Tr}\big[e^{-\beta\mathcal{H}}\underbrace{e^{\imath\mathcal{H}t/\hbar}Ae^{-\imath\mathcal{H}t/\hbar}}_{A(t)}A\big]$	$e^{\imath E_0 t/\hbar}\big\langle\phi_0\|A\underbrace{e^{-\imath\mathcal{H}t/\hbar}A\|\phi_0\big\rangle}_{\|\psi(t)\rangle}$