Orthogonal Expansion of Dynamical Variables

\[ A(t) = \sum_{k=0}^{\infty} C_k(t) |f_k\rangle. \]

(1)

**Step #1: [M.H. Lee]**

- Orthogonal basis, \(|f_0\rangle, |f_1\rangle, \ldots\), with initial condition, \(|f_0\rangle = A(0)\).
- Quantum statistics: \(|f_k\rangle\) form orthogonal set of operators.
- Classical statistics: \(|f_k\rangle\) form orthogonal set of phase-space functions.
- Generation of orthogonal directions: \( \langle f_k | iL | f_k \rangle = 0 \).

Recurrence relations for basis vectors \(|f_k\rangle\):

\[ |f_{k+1}\rangle = iL|f_k\rangle + \Delta_k|f_{k-1}\rangle, \quad \Delta_{k+1} = \frac{\langle f_{k+1} | f_{k+1} \rangle}{\langle f_k | f_k \rangle}, \quad k = 0, 1, 2, \ldots \]  

(2)

Conditions: \(|f_{-1}\rangle \equiv 0\), \(|f_0\rangle \equiv A\), \(\Delta_0 \equiv 0\).

First three iterations spelled out in [nl83].

**Step #2: [M.H. Lee]**

- Time-dependent coefficients of basis vectors: \(C_k(t)\).
- Substitute (1) into equation of motion from [nl81]: \(dA/dt = iLA\).
- \(d/dt\) acts on \(C_k(t)\) and \(L\) acts on \(|f_k\rangle\).

Comparison of coefficients in

\[ \sum_{k=0}^{\infty} \dot{C}_k(t) |f_k\rangle = \sum_{k=0}^{\infty} C_k(t) \left[ |f_{k+1}\rangle - \Delta_k|f_{k-1}\rangle \right] \]  

(3)

yields set of coupled, linear, first-order ODEs for functions \(C_k(t)\):

\[ \dot{C}_k(t) = C_{k-1}(t) - \Delta_{k+1}C_{k+1}(t), \quad k = 0, 1, 2, \ldots \]  

(4)

Conditions: \(C_{-1}(t) \equiv 0\), \(C_k(0) = \delta_{k,0}, \quad k = 0, 1, 2, \ldots \)

Normalized fluctuation function (see [nl39]):

\[ C_0(t) = \frac{\langle A(t) | A(0) \rangle}{\langle A(0) | A(0) \rangle} = \frac{\tilde{\Phi}(t)}{\Phi(0)}. \]  

(5)