Orthogonal Expansion of Dynamical Variables [nln82]

$$A(t) = \sum_{k=0}^{\infty} C_k(t)|f_k\rangle. \tag{1}$$

Step #1: [M.H. Lee]

- Orthogonal basis, $|f_0\rangle, |f_1\rangle, \ldots$, with initial condition, $|f_0\rangle = A(0)$.
- Quantum statistics: $|f_k\rangle$ form orthogonal set of operators.
- Classical statistics: $|f_k\rangle$ form orthogonal set of phase-space functions.
- Generation of orthogonal directions: $\langle f_k|\imath L|f_k\rangle = 0$.

Recurrence relations for basis vectors $|f_k\rangle$:

$$|f_{k+1}\rangle = \imath L|f_k\rangle + \Delta_k|f_{k-1}\rangle, \quad \Delta_{k+1} = \frac{\langle f_{k+1}|f_{k+1}\rangle}{\langle f_k|f_k\rangle}, \quad k = 0, 1, 2, \ldots \tag{2}$$

Conditions: $|f_{-1}\rangle \doteq 0, \quad |f_0\rangle \doteq A, \quad \Delta_0 \doteq 0.$

First three iterations spelled out in [nln83].

Step #2: [M.H. Lee]

- Time-dependent coefficients of basis vectors: $C_k(t)$.
- Substitute (1) into equation of motion from [nln81]: $dA/dt = \imath LA$.
- d/dt acts on $C_k(t)$ and L acts on $|f_k\rangle$.

Comparison of coefficients in

$$\sum_{k=0}^{\infty} \dot{C}_k(t)|f_k\rangle = \sum_{k=0}^{\infty} C_k(t)\big[\underbrace{|f_{k+1}\rangle - \Delta_k|f_{k_1}|\rangle}_{\imath L|f_k\rangle}\big] \tag{3}$$

yields set of coupled, linear, first-order ODEs for functions $C_k(t)$:

$$\dot{C}_k(t) = C_{k-1}(t) - \Delta_{k+1}C_{k+1}(t), \quad k = 0, 1, 2, \ldots \tag{4}$$

Conditions: $C_{-1}(t) \equiv 0, \quad C_k(0) = \delta_{k,0}, \quad k = 0, 1, 2, \ldots$

Normalized fluctuation function (see [nln39]):

$$C_0(t) = \frac{\langle A(t)|A(0)\rangle}{\langle A(0)|A(0)\rangle} = \frac{\tilde{\Phi}(t)}{\tilde{\Phi}(0)}. \tag{5}$$