Link to Generalized Langevin Equation [nln86]

Define two functions from the $c_k(z)$ introduced in [nln84]:

$$\Sigma(z) \doteq \Delta_1 \frac{c_1(z)}{c_0(z)}, \quad b_k(z) \doteq \frac{c_k(z)}{c_0(z)}. \tag{1}$$

Rewrite algebraic equations (2) for $k = 0$ of [nln84] using $\Sigma(z)$ and $b_k(z)$:

$$zc_0(z) + \Sigma(z)c_0(z) = 1, \tag{2a}$$

$$zc_k(z) + \Sigma(z)c_k(z) = b_k(z), \quad k = 1, 2, \ldots \tag{2b}$$

Inverse Laplace transforms of these functions then satisfy

$$\dot{C}_0(t) + \int_0^t dt' \tilde{\Sigma}(t - t')C_0(t') = 0, \tag{3a}$$

$$\dot{C}_k(t) + \int_0^t dt' \tilde{\Sigma}(t - t')C_k(t') = B_k(t), k = 1, 2, \ldots \tag{3b}$$

Recall orthogonal expansion (1) in [nln82] of dynamical variable:

$$A(t) = \sum_{k=0}^{\infty} C_k(t)|f_k\rangle. \tag{4}$$

From (3) and (4) follows generalized Langevin equation [M. H. Lee 1983]:[1]

$$\dot{A}(t) + \int_0^{\infty} dt' \tilde{\Sigma}(t - t')A(t') = F(t). \tag{5}$$

Orthogonal exapnsion of random force:

$$F(t) = \sum_{k=1}^{\infty} B_k(t)|f_k\rangle, \quad B_k(0) = \delta_{k,1}. \tag{6}$$

Absence of correlations between dynamical variable and random force:

$$\langle F(t)|A(0)\rangle = \sum_{k=1}^{\infty} B_k(t)\langle f_k|f_0\rangle = 0. \tag{7}$$

Fluctuation-dissipation relation between $\tilde{\Sigma}(t)$ (memory function) and $F(t)$:

$$\langle F(t)|F(0)\rangle = B_1(t)\langle f_1|f_1\rangle = \Delta_1 B_1(t)\langle f_0|f_0\rangle = \tilde{\Sigma}(t)\langle f_0|f_0\rangle. \tag{8}$$

[1] Lower integration boundary is specific to initial-value problem under consideration.