

n-Pole Approximation [nl87]

Relaxation function after n successive projections (Mori formalism [nl36]) or n iterations (recursion method [nl84]),

$$c_0(z) = \frac{1}{z + \frac{\Delta_1}{z + \frac{\Delta_2}{z + \dots + \frac{\Delta_{n-1}}{z + \frac{1}{\tau_n}}}}}, \quad (1)$$

has termination function represented by a relaxation time τ_n :

$$\Gamma_n(z) = \Delta_n \frac{c_n(z)}{c_{n-1}(z)} = \frac{1}{\tau_n}. \quad (2)$$

Consequences:

- Singularity structure of $c_0(z)$ reduced to n poles in complex z -plane.
- Frequency moments $M_{2k} = \langle \omega^{2k} \rangle$ divergent for $k \geq n$.

Limiting cases:

- $\tau_n \rightarrow \infty$:
All poles approach ω -axis: $z = \epsilon - i\omega$ with $\epsilon \rightarrow 0$.
Spectral density $\Phi_0(\omega)$ is a sum of δ -functions.
- $\tau_n \rightarrow 0$:
Transition to $(n-1)$ -pole approximation: $\frac{\Delta_{n-1}}{z + 1/\tau_n} \rightarrow \frac{1}{\tau_{n-1}}$.

Illustrations for $n = 1, 2, 3$:

- relaxation function: $c_0(z) = \frac{1}{z + \Sigma(z)}$,
- memory function: $\Sigma(z)$,
- spectral density: $\Phi_0(\omega) = 2 \lim_{\epsilon \rightarrow 0} \text{Re}[c_0(\epsilon - i\omega)]$.

1-pole spectral approximation

Memory function representing white noise:

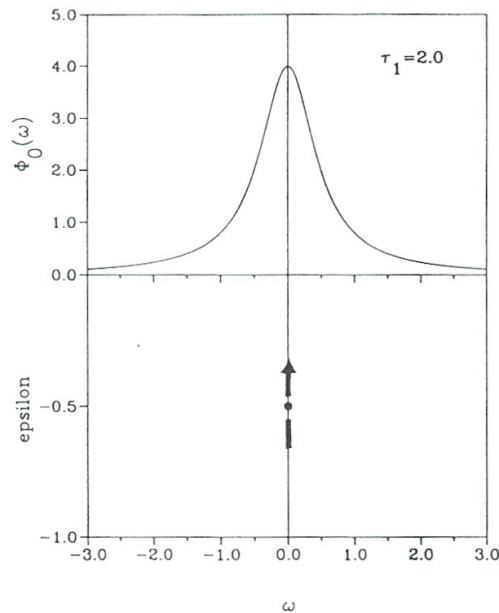
$$\Sigma(z) = \frac{1}{\tau_1} = \text{const.}$$

Relaxation function:

$$c_0(z) = \frac{1}{z + \frac{1}{\tau_1}} = \frac{\tau_1}{\tau_1 z + 1}.$$

Spectral density:

$$\Phi_0(\omega) = \frac{2\tau_1}{1 + \tau_1^2 \omega^2}.$$



2-pole spectral approximation

Memory function representing classical relaxator:

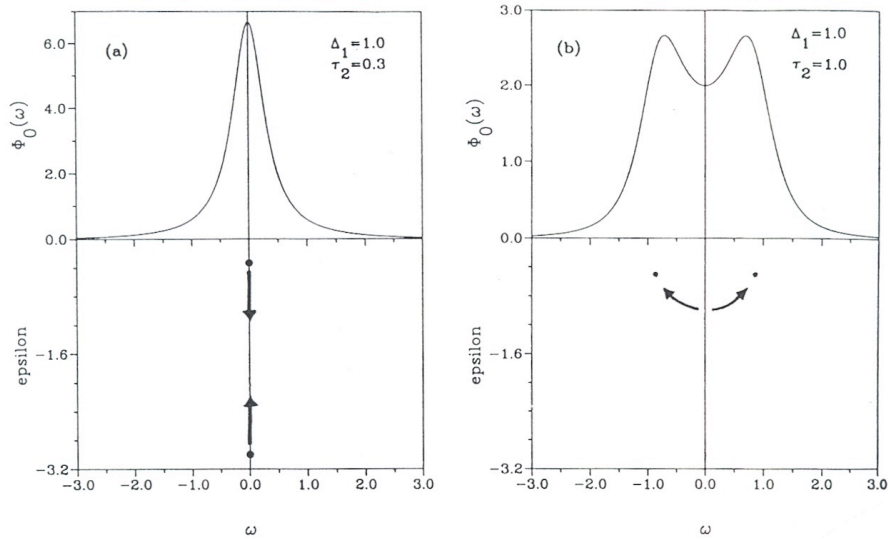
$$\Sigma(z) = \frac{\Delta_1}{z + \frac{1}{\tau_1}}.$$

Relaxation function:

$$c_0(z) = \frac{\tau_2 z + 1}{\tau_2 z^2 + z + \Delta_1 \tau_2}.$$

Spectral density:

$$\Phi_0(\omega) = \frac{2\Delta_1 \tau_2}{\tau_2^2 (\Delta_1 - \omega^2)^2 + \omega^2}.$$



3-pole spectral approximation

Memory function representing damped classical oscillator:

$$\Sigma(z) = \frac{\Delta_1}{z + \frac{\Delta_2}{z + \frac{1}{\tau_3}}}.$$

Relaxation function:

$$c_0(z) = \frac{\tau_3 z^2 + z + \Delta_2 \tau_3}{\tau_3 z^3 + z^2 + (\Delta_1 + \Delta_2) \tau_3 z + \Delta_1}.$$

Spectral density:

$$\Phi_0(\omega) = \frac{2\Delta_1 \Delta_2 \tau_3}{[\tau_3 \omega(\omega^2 - \Delta_1 - \Delta_2)]^2 + (\omega^2 - \Delta_1)^2}.$$

