Magnetic Resonance or Scattering [nln97]

Magnetic probe: $\mathcal{H}_1(t) = -\mathbf{M} \cdot \mathbf{h}(t)$. We set $\hbar = 1$ throughout.

Linear response: $\langle M_\mu(\mathbf{r}, t)\rangle - \langle M_\mu\rangle_{\rm eq} = \sum_\nu \int d^3r' \int dt\, \tilde{\chi}_{\mu\nu}(\mathbf{r} - \mathbf{r}', t - t')h_\nu(\mathbf{r}', t')$.

Response function: $\tilde{\chi}_{\mu\nu}(\mathbf{r}, t) = \imath\theta(t)\langle[M_\mu(\mathbf{r}, t), M_\nu(\mathbf{0}, 0)]\rangle = \imath\theta(t)[S^\mu_{\mathbf{l}+\mathbf{r}}(t), S^\nu_{\mathbf{l}}]\rangle$.

Generalized susceptibility: $\chi_{\mu\nu}(\mathbf{q}, \omega) = \sum_{\mathbf{r}} e^{\imath\mathbf{q}\cdot\mathbf{r}} \int_{-\infty}^{+\infty} dt\, e^{\imath\omega t}\tilde{\chi}_{\mu\nu}(\mathbf{r}, t)$.

Correlation function: $\tilde{S}_{\mu\nu}(\mathbf{r}, t) = \langle S^\mu_{\mathbf{l}+\mathbf{r}}(t)S^\nu_{\mathbf{l}}\rangle$.

Dynamic structure factor: $S_{\mu\nu}(\mathbf{q}, \omega) = \sum_{\mathbf{r}} e^{\imath\mathbf{q}\cdot\mathbf{r}} \int_{-\infty}^{+\infty} dt\, e^{\imath\omega t}\tilde{S}_{\mu\nu}(\mathbf{r}, t)$.

Relation from [nln39]: $S_{\mu\nu}(\mathbf{q}, \omega) = \dfrac{2\chi''_{\mu\nu}(\mathbf{q}, \omega)}{1 - e^{-\beta\omega}}$.

Experimental techniques:

- Ferromagnetic resonance, EPR.
  - Long wavelengths (long compared to lattice spacing) probed.
  - Relevant quantitity: $\chi''_{\mu\nu}(\mathbf{q} \simeq 0, \omega)$.
- Inelastic neutron scattering.
  - Interaction with magnetic dipole moment of neutron.
  - Momentum transfer $\mathbf{q}$ and energy transfer ω of neutrons well matched with energy-momentum relations $\epsilon(\mathbf{q})$ of typical collective magnetic excitations.
  - Scattering cross section: $\dfrac{d^2\sigma}{d\omega d\Omega} \propto S_{\mu\nu}(\mathbf{q}, \omega)$.
- Nuclear magnetic resonance, NMR.
  - Localized probe (nuclear magnetic moment) interacts with electronic magnetism in immediate vicinity.
  - Spin-lattice relaxation rate: $\dfrac{1}{T_1} \propto \sum_{\mathbf{q}} S_{\mu\nu}(\mathbf{q}, \omega_{\rm N})$.
  - Nuclear Larmor frequency $\omega_{\rm N}$ is very small compared to typical electronic magnetic excitations.