Spectral Lines from Finite $\Delta_k$-Sequences

Orthogonal expansion in [nl83] comes natural stop: $|f_{K+1}| = 0$.
Relaxation function has $K + 1$ poles on imaginary $z$-axis.

$$
\Rightarrow c_0(z) = \frac{1}{z + \frac{\Delta_1}{z + \frac{\Delta_2}{z + \cdots + \frac{\Delta_K}{z}}}} = \frac{p_K(z)}{q_{K+1}(z)}.
$$

Spectral density: $\Phi_0(\omega) = \pi \sum_{l=1}^{L} u_l \left[ \delta(\omega - \omega_l) + \delta(\omega + \omega_l) \right]$. 

- $K$: number of nonzero continued-fraction coefficients $\Delta_k$.
- $L$: number of spectral lines (with frequencies $\omega_l$).
- Relation: $K = \left\{ \begin{array}{ll}
2L - 1 \text{ (odd)} & \text{if all } \omega_l > 0, \\
2L - 2 \text{ (even)} & \text{if } \omega_l = 0 \text{ occurs.}
\end{array} \right.$

Special case $u_1 = u_2$ for $L = 2$:

$$
\Delta_1 = \frac{1}{2}(\omega_1^2 + \omega_2^2), \quad \Delta_2 = \Delta_1 - \Delta_3, \quad \Delta_3 = \frac{2\omega_1^2\omega_2^2}{\omega_1^2 + \omega_2^2}.
$$

Limit $\omega_1 \to 0$ implies $\Delta_3 \to 0$ and $\Delta_2 \to \Delta_1$. 