Spectral Lines from Finite Δ_k-Sequences [nln99]

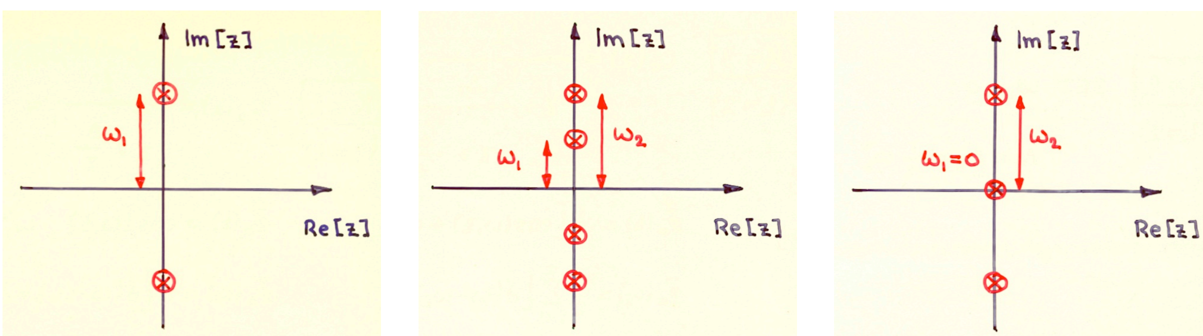
Orthogonal expansion in [nln83] comes natural stop: $|f_{K+1}\rangle = 0$.

Relaxation function has $K+1$ poles on imaginary z-axis.

$$\Rightarrow c_0(z) = \cfrac{1}{z + \cfrac{\Delta_1}{z + \cfrac{\Delta_2}{z + \cdots \cdots + \cfrac{\Delta_K}{z}}}} = \frac{p_K(z)}{q_{K+1}(z)}.$$

Spectral density: $\Phi_0(\omega) = \pi \sum_{l=1}^{L} u_l \big[\delta(\omega - \omega_l) + \delta(\omega + \omega_l)\big]$.

- K: number of nonzero continued-fraction coefficients Δ_k.
- L: number of spectral lines (with frequencies ω_l).
- Relation: $K = \begin{cases} 2L-1 \text{ (odd)} & \text{if all } \omega_l > 0, \\ 2L-2 \text{ (even)} & \text{if } \omega_l = 0 \text{ occurs.} \end{cases}$



Special case $u_1 = u_2$ for $L = 2$:

$$\Delta_1 = \frac{1}{2}(\omega_1^2 + \omega_2^2), \quad \Delta_2 = \Delta_1 - \Delta_3, \quad \Delta_3 = \frac{2\omega_1^2\omega_2^2}{\omega_1^2 + \omega_2^2}.$$

Limit $\omega_1 \to 0$ implies $\Delta_3 \to 0$ and $\Delta_2 \to \Delta_1$.