**Predator-Prey System**

A population of foxes (predator $F$) feeds on a population of hares (prey $H$). The birth rate of foxes is proportional to the fox population and to the amount of food available. Foxes die naturally, i.e. at a rate proportional to the fox population. Hares die primarily through encounters with foxes and are born at a rate proportional to the hare population.

**Deterministic time evolution: Lotka-Volterra model.**

\[
\frac{dH}{dt} = aH - bHF, \quad \frac{dF}{dt} = bHF - dF.
\]

Anharmonic oscillation.

Hyperbolic fixed point at $(0,0)$.

Elliptic fixed point at $(d/b, a/b)$.

Fluctuations excluded.

Environmental effects in initial conditions only.

**Stochastic time evolution: master equation.**

\[
\frac{\partial}{\partial t} P(H, F, t) = \sum_{H', F'} \left[ W(H|H'; F|F') P(H', F', t) - W(H'|H; F'|F) P(H, F, t) \right],
\]

Non-vanishing transition rates:

- $W(H + 1|H; F|F) = aH$ (prey is born)
- $W(H - 1|H; F + 1|F) = bHF$ (predator thrives on prey)
- $W(H|H; F - 1|F) = dF$ (predator dies)

Fluctuations now included.

Environmental effects in initial conditions and in contingencies of stochastic time evolution.
- Time evolution of deterministic system: Lotka-Volterra model

![Graph of Lotka-Volterra model](image)

- Computer simulation of stochastic system: master equation

![Graph of master equation simulation](image)

- Observation of real system: two species of mites

![Graph of real system observation](image)

[images from Gardiner 1985]