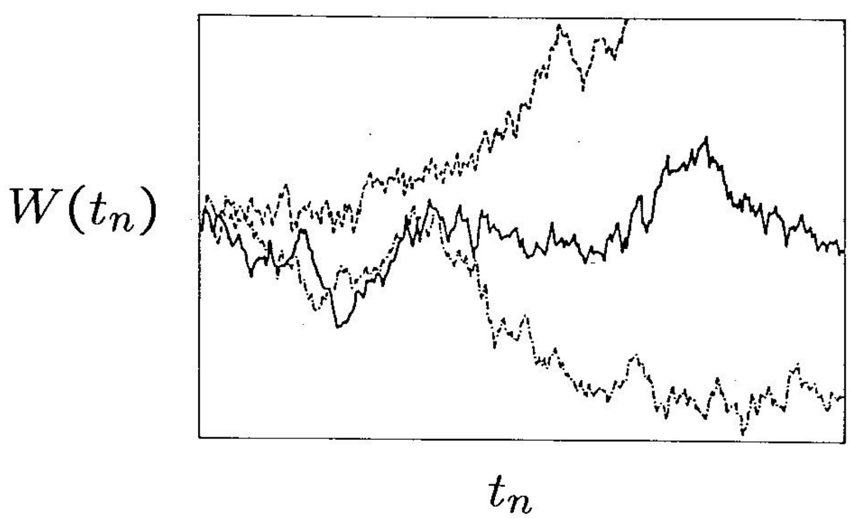
Wiener process [nls4]

Specifications:

1. For $t_0 < t_1 < \cdots$, the position increments $\Delta x(t_n, t_{n-1}), n = 1, 2, \ldots$ are independent random variables.
2. The increments depend only on time differences: $\Delta x(t_n, t_{n-1}) = \Delta x(dt_n)$, where $dt_n = t_n - t_{n-1}$.
3. The increments satisfy the Lindeberg condition:
$$\lim_{dt \to 0} \frac{1}{dt} \mathcal{P}[\Delta x(dt) \geq \epsilon] = 0 \text{ for all } \epsilon > 0$$

The diffusion process discussed previously [nex26], [nex27], [nex97],
$$P(x, t + dt|x_0, t) = \frac{1}{\sqrt{4\pi D dt}} \exp\left(-\frac{(x - x_0)^2}{4D dt}\right),$$
is, for $dt \to 0$, a realization of the Wiener process. Sample paths of the Wiener process thus realized are everywhere continuous and nowhere differentiable.

$$W(t_n) = \sum_{i=1}^{n} \Delta x(dt_i), \qquad t_n = \sum_{i=1}^{n} dt_i.$$



[adapted from Gardiner 1985]