Fokker-Planck equation with constant coefficients

Convert the Fokker-Planck equation with constant coefficients of drift and diffusion,

\[
\frac{\partial}{\partial t} P(x, t|x_0) = -A \frac{\partial}{\partial x} P(x, t|x_0) - \frac{1}{2} B \frac{\partial^2}{\partial x^2} P(x, t|x_0),
\]

into an ordinary differential equation for the characteristic function,

\[
\Phi(k, t) = \int_{-\infty}^{+\infty} dx \ e^{ikx} P(x, t|x_0).
\]

(a) Solve this differential equation (by elementary means) and infer \( P(x, t|x_0) \) via inverse Fourier transform. Use the initial condition \( P(x, 0|x_0) = \delta(x - x_0) \).

(b) Identify the mean \( \langle x \rangle \) and the variance \( \langle x^2 \rangle \) in the solution \( P(x, t|x_0) \).

(c) Simplify the solution \( P(x, t|x_0) \) for the special case \( B = 0 \) (no diffusion).

Solution: