[nex101] Fokker-Planck equation with constant coefficients

Convert the Fokker-Planck equation with constant coefficients of drift and diffusion,

$$\frac{\partial}{\partial t}P(x,t|x_0) = -A\frac{\partial}{\partial x}P(x,t|x_0) - \frac{1}{2}B\frac{\partial^2}{\partial x^2}P(x,t|x_0),$$

into an ordinary differential equation for the characteristic function,

$$\Phi(k,t) \doteq \int_{-\infty}^{+\infty} dx\, e^{ikx} P(x,t|x_0).$$

(a) Solve this differential equation (by elementary means) and infer $P(x,t|x_0)$ via inverse Fourier transform. Use the initial condition $P(x,0|x_0) = \delta(x - x_0)$.
(b) Identify the mean $\langle\langle x\rangle\rangle$ and the variance $\langle\langle x^2\rangle\rangle$ in the solution $P(x,t|x_0)$.
(c) Simplify the solution $P(x,t|x_0)$ for the special case $B = 0$ (no diffusion).

Solution: