Life expectancy of the young and the old

The distribution of lifetimes in some population is \( f(t) = (4t/T^2)e^{-2t/T} \).

(a) Show that \( f(t) \) is properly normalized and that the parameter \( T \) is the average lifetime of individuals.

(b) Calculate the conditional probability distribution \( P_c(t|\tau) \) for the remaining lifetime of individuals of age \( \tau \). Use the expression constructed in [nex38].

(c) If we define the life expectancy \( T_\tau \) as the average remaining lifetime for an individual of age \( \tau \) calculate \( T_\tau \) as a function of \( T \) and \( \tau \).

(d) What is the life-expectancy ratio \( T_\infty/T_0 \) of the very old and the very young.

Solution: