[nex110] Air in leaky tank IV: evolution of mean and variance

At time $t = 0$ a tank of volume V contains n_0 molecules of air (disregarding chemical distinctions). The tank has a tiny leak and exchanges molecules with the environment, which has a constant density ρ of air molecules. Given the transition rate $W(m|n)$ and generating function $G(z,t)$ derived in [nex48] for this process we are in a position to calculate the the time evolution of the mean $\langle\langle n(t)\rangle$ and the variance $\langle\langle n^2(t)\rangle$ in three different ways with moderate effort.
(a) Calculate $\langle n(t)\rangle_f$ and $\langle n^2(t)\rangle_f$ directly from the (known) $G(z,t)$ and infer mean and variance from these factorial moments.
(b) Infer equations of motions for the $\langle n^m(t)\rangle_f$, $m = 1, 2, \ldots$ from the (known) PDE for $G(z,t)$. Solve these equations for $m = 1, 2$. In this system a closed set of equations results for any m.
(c) Calculate the jump moments $\alpha_l(m) = \sum_n (n-m)^l W(n|m)$ from the known transition rates and show that the associated equations of motion $d\langle n\rangle/dt = \langle\alpha_1(n)\rangle$, $d\langle n^2\rangle/dt = \langle\alpha_2(n)\rangle + 2\langle n\alpha_1(n)\rangle$ for the first two moments are equivalent to those previously derived for the factorial moments.

Solution: