At time $t = 0$ a tank of volume $V$ contains $n_0$ molecules of air (disregarding chemical distinctions). The tank has a tiny leak and exchanges molecules with the environment, which has a constant density $\rho$ of air molecules. Given the transition rate $W(m|n)$ and generating function $G(z,t)$ derived in [nex48] for this process we are in a position to calculate the the time evolution of the mean $\langle n(t) \rangle$ and the variance $\langle n^2(t) \rangle$ in three different ways with moderate effort.

(a) Calculate $\langle n(t) \rangle_f$ and $\langle n^2(t) \rangle_f$ directly from the (known) $G(z,t)$ and infer mean and variance from these factorial moments.

(b) Infer equations of motions for the $\langle n^m(t) \rangle_f$, $m = 1, 2, \ldots$ from the (known) PDE for $G(z,t)$. Solve these equations for $m = 1, 2$. In this system a closed set of equations results for any $m$.

(c) Calculate the jump moments $\alpha_l(m) = \sum_n (n-m)^l W(n|m)$ from the known transition rates and show that the associated equations of motion $d\langle n \rangle/dt = \langle \alpha_1(n) \rangle$, $d\langle n^2 \rangle/dt = \langle \alpha_2(n) \rangle + 2\langle n\alpha_1(n) \rangle$ for the first two moments are equivalent to those previously derived for the factorial moments.

Solution: