[nex116] Modified linear birth rate IV: probability distribution

Consider a linear birth-death process with a modified birth rate to be used in the master equation:

$$W(m|n) = \lambda(n+1)\delta_{m,n+1} + \mu n\delta_{m,n-1}, \quad \lambda < \mu.$$

In [nex44] we had shown that the original linear birth rate $T_+(n) = n\lambda$ leads to the extinction of the population. In [nex113] we showed that the modified linear birth rate $T_+(n) = (n+1)\lambda$ leads to a nonvanishing stationary distribution $P_s(n)$: the Pascal distribution. In [nex115] we have calculated the generating function $G(z,t)$ for the case of zero initial population. Use that result here to calculate the explicit result,

$$P(n,t) = \frac{\mu - \lambda}{\mu - \lambda e^{(\lambda-\mu)t}} \left(\frac{\lambda[1 - e^{(\lambda-\mu)t}]}{\mu - \lambda e^{(\lambda-\mu)t}} \right)^n,$$

for the time-dependence of the probability distribution $P(n,t)$. Check the limit $t \to 0$ to verify the imposed initial condition and the limit $t \to \infty$ to confirm the result of $P_s(n)$ previously obtained by other means.

Solution: