[nex121] Brownian harmonic oscillator I: Fourier analysis

The Brownian harmonic oscillator is specified by the Langevin-type equation,

$$m\ddot{x} + \gamma\dot{x} + kx = f(t), \tag{1}$$

where m is the mass of the particle, γ represents attenuation without memory, $k = m\omega_0^2$ is the spring constant, and $f(t)$ is a white-noise random force. Convert the ODE (1) into a an algebraic equation for the Fourier amplitude $\tilde{x}(\omega)$ of the position and the Fourier amplitude $\tilde{f}(\omega)$ of the random force. Proceed as in [nex119] to infer the spectral density

$$S_{xx}(\omega) = \frac{2\gamma k_B T}{m^2(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}.$$

of the position coordinate. In the process use the result $S_{ff}(\omega) = 2k_B T\gamma$ for the random-force spectral density as dictated by the fluctuation-dissipation theorem.

Solution: