The Brownian harmonic oscillator is specified by the Langevin-type equation,

\[ m\ddot{x} + \gamma \dot{x} + kx = f(t), \tag{1} \]

where \( m \) is the mass of the particle, \( \gamma \) represents attenuation without memory, \( k = m\omega_0^2 \) is the spring constant, and \( f(t) \) is a white-noise random force. Convert the ODE (1) into an algebraic equation for the Fourier amplitude \( \tilde{x}(\omega) \) of the position and the Fourier amplitude \( \tilde{f}(\omega) \) of the random force. Proceed as in [nex119] to infer the spectral density

\[ S_{xx}(\omega) = \frac{2\gamma k_B T}{m^2(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}. \]

of the position coordinate. In the process use the result \( S_{ff}(\omega) = 2k_B T\gamma \) for the random-force spectral density as dictated by the fluctuation-dissipation theorem.

Solution: