

[nex124] Combinatorics of poker hands

The game of poker is played with a deck of 52 cards: 4 suits ($\clubsuit, \diamondsuit, \heartsuit, \spadesuit$) with 13 ranks each (2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A). The table below states the number of hands of five cards from nine different types and the probability of each type. Explain how you arrive at each number in the middle column.

type of hand	number	probability
straight flush	40	0.000015
four of a kind	624	0.000240
full house	3744	0.001441
flush	5108	0.001965
straight	10,200	0.003925
three of a kind	54,912	0.021129
two pairs	123,552	0.047539
one pair	1,098,240	0.422569
nothing	1,302,540	0.501177
total	2,598,960	1.000000

A *straight flush* has five cards in sequence, all from the same suit. A *straight* has five cards of sequential rank, not all from the same suit. A *full house* has three of one kind (rank) and a pair of a different rank. A *flush* has five cards of the same suit but not all in sequence. In a *straight* (*flush* or not) the ace (A) can be the highest or the lowest card in rank.

Solution: