

### [nex126] Cumulants expressed in terms of moments

The characteristic function  $\Phi_X(k)$  of a probability distribution  $P_X(x)$ , obtained via Fourier transform as described in [nl47], can be used to generate the moments  $\langle X^n \rangle$  and the cumulants  $\langle\langle X^n \rangle\rangle$  via the expansions

$$\Phi_X(k) = \sum_{n=0}^{\infty} \frac{(ik)^n}{n!} \langle X^n \rangle, \quad \ln \Phi_X(k) = \sum_{n=1}^{\infty} \frac{(ik)^n}{n!} \langle\langle X^n \rangle\rangle.$$

Use these relations to express the first four cumulants in terms of the first four moments. The results are stated in [nl47]. Describe your work in some detail.

**Solution:**