[nex127] Sums of independent exponentials

Consider n independent random variable $X_1, \ldots, X_n$ with range $x_i \geq 0$ and identical exponential distributions,

$$P_1(x_i) = \frac{1}{\xi}\, e^{-x_i/\xi}, \quad i = 1, \ldots, n.$$

Use the transformation relation from [nln49],

$$P_2(x) = \int dx_1 \int dx_2\, P_1(x_1)P_1(x_2)\delta(x - x_1 - x_2) = \int dx_1 P_1(x_1)P_1(x - x_1),$$

inductively to calculate the probability distribution $P_n(x)$, $n \geq 2$ of the stochastic variable

$$X = X_1 + \cdots + X_n.$$

Find the mean value $\langle X \rangle$, the variance $\langle\langle X^2 \rangle\rangle$, and the value x_p where $P_n(x)$ has its peak value.

Solution: