[nex128] Release of Brownian particle from box confinement

Consider a physical ensemble of Brownian particles uniformly distributed inside a one-dimensional box. The initial density is

$$\rho(x,0) = \frac{1}{2}\theta\big(1-|x|\big),$$

where $\theta(x)$ is the step function. At time $t=0$ the particles are released to diffuse left and right. Use the two methods presented in [nln73] to calculate the analytic solution,

$$\rho(x,t) = \frac{1}{4}\left[\mathrm{erf}\left(\frac{x+1}{\sqrt{4Dt}}\right) - \mathrm{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right)\right],$$

of the diffusion equation, where the error function is defined as follows:

$$\mathrm{erf}(x) \doteq \frac{2}{\sqrt{\pi}}\int_0^x du\, e^{-u^2}.$$

(a) In the Fourier analysis of [nln73] first calculate the initial Fourier amplitudes via (2) and then use the result in the integration (3).
(b) In the Green's function analysis of [nln73] perform the involution integral (5) with the point-source solution (4) and the initial rectangular initial distribution pertaining to this application.
(c) Plot $\rho(x,t)$ versus x for $-3 \leq x \leq +3$ and $Dt = 0, 0.04, 0.2, 1, 5$.

Solution: