Random bus schedules.

Three bus companies A, B, C offer schedules in the form of a probability density $f(t)$ for the intervals between bus arrivals at the bus stop:

\[ A : \quad f(t) = \delta(t - T), \quad B : \quad f(t) = \frac{1}{T} e^{-t/T}, \quad C : \quad f(t) = \frac{4t}{T^2} e^{-2t/T}. \]

(i) Find the probability $P_0(t)$ that the interval between bus arrivals is larger than $t$.
(ii) Find the mean time interval $\tau_B$ between bus arrivals and the variance thereof.
(iii) Find the probability $Q_0(t)$ that no arrivals occur in a randomly chosen time interval $t$.
(iv) Find the probability density $g(t)$ of the time a passenger waits for the next bus from the moment he/she arrives at the bus stop.
(v) Find the average waiting time $\tau_P$ of passengers and the variance thereof.

Solution: