Pascal distribution.

Consider the quantum harmonic oscillator in thermal equilibrium at temperature $T$. The energy levels (relative to the ground state) are $E_n = n \hbar \omega$, $n = 0, 1, 2, \ldots$

(a) Show that the system is in level $n$ with probability

$$P(n) = (1 - \gamma)^n \gamma^n, \quad \gamma = \exp(-\hbar \omega / k_B T).$$

$P(n)$ is called Pascal distribution or geometric distribution.

(b) Calculate the factorial moments $\langle n^m \rangle_f$ and the factorial cumulants $\langle \langle n^m \rangle \rangle_f$ of this distribution.

(c) Show that the Pascal distribution has a larger variance $\langle \langle n^2 \rangle \rangle$ than the Poisson distribution with the same mean value $\langle n \rangle$.

Solution: