[nex22] Pascal distribution.

Consider the quantum harmonic oscillator in thermal equilibrium at temperature T. The energy levels (relative to the ground state) are $E_n = n\hbar\omega$, $n = 0, 1, 2, \ldots$
(a) Show that the system is in level n with probability

$$P(n) = (1-\gamma)\gamma^n, \quad \gamma = \exp(-\hbar\omega/k_BT).$$

$P(n)$ is called *Pascal* distribution or *geometric* distribution.
(b) Calculate the factorial moments $\langle n^m \rangle_f$ and the factorial cumulants $\langle\langle n^m \rangle\rangle_f$ of this distribution.
(c) Show that the Pascal distribution has a larger variance $\langle\langle n^2 \rangle\rangle$ than the Poisson distribution with the same mean value $\langle n \rangle$.

Solution: